ÉRETTSÉGI VIZSGA ● 2010. május 4.

MATEMATIKA ANGOL NYELVEN

EMELT SZINTŰ ÍRÁSBELI VIZSGA

2010. május 4. 8:00

Az írásbeli vizsga időtartama: 240 perc

| Pótlapok száma	
Tisztázati	
Piszkozati	

OKTATÁSI ÉS KULTURÁLIS MINISZTÉRIUM

Important information

1. The exam is 240 minutes long, after that you should stop working.

2. You may solve the problems in any order.

3. In Section **II**, you are only required to solve four out of the five problems**. Please remember to enter the number of the question you have not attempted into the empty square below.** Should there *arise any ambiguity* for the examiner whether the question is to be marked or not, it is question no. 9 that will not going to be assessed.
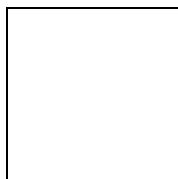
4. You may work with any calculator as long as it is not capable of storing and displaying textual information and you may also consult any type of four digit mathematical table. The use of any other kind of electronic device or written source is forbidden.

5. **Remember to show your reasoning, because a major part of the score is given for this component of your work.**

6. **Remember to outline the substantial calculations**.

7. When you refer to a theorem that has been covered at school and has a common name (e.g. Pythagoras' theorem, sine rule, etc.) you are not expected to state it meticulously; it is usually sufficient to put the name of the theorem, however, you are supposed to show why does it apply. Any reference to any other theorem, however, can be accepted only if it is stated exactly with all the conditions (proof is not required) and you explain how it applies in the given situation.

8. Remember to answer each question (i.e. providing the result) also in textual form.

9. You are supposed to work in pen; diagrams, however, may also be drawn in pencil. Anything written in pencil outside the diagrams cannot be evaluated by the examiner. Any solution or some part of a solution that is crossed out will not be marked.

10. There is only one solution will be marked for every question. If you attempt a question more than once then you should **clearly indicate** the one to be marked.

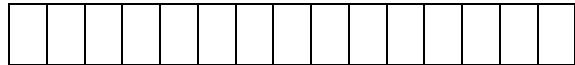11. Please, do not write anything in the shaded rectangular areas.

I.

1.

a) Solve the following system of equations on the set of pairs of real numbers.

$$\begin{cases} \log_2\left(xy^3\right) = 1 \\ \log_2\left(x^2y\right) = -3 \end{cases}$$

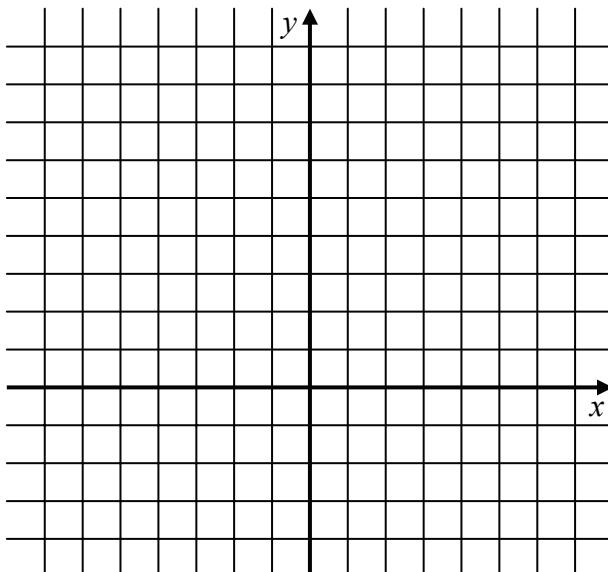**b)** Find all those positive integer values of k for which $\log_{3^k} 729$ is a positive integer.

a)	7 points	
b)	5 points	
T.:	12 points	

2. **a)** Prove that the quadrilateral whose vertices are the points $A(0; 1)$, $B(4; 2)$, $C(3; 6)$
and $D(-5; 4)$ is a trapezium.

b) Kate was drawing a simple complete graph with 253 edges altogether in such a
way that the four vertices A, B, C, D of the trapezium were among its vertices.
How many new vertices of the graph did she need to draw for that?
At most how many edges of this complete graph may be deleted if one still wants
it to remain connected?

a)	4 points	
b)	8 points	
T.:	12 points	

3. An airline company is operating a direct flight between two European cities. The flights are leaving if there are at least 10 passengers and no plane can carry more than 36 passengers. Looking for improving the utilization of its flights the company is considering the following option: in case of 20 or less passengers the price of the ticket is 16 000 Ft per passenger. If, however, there are more than 20 passengers then the price is calculated as follows: the difference betwen the actual number of passengers and 20 is multiplied by 400 and the originally 16 000 Ft fare is decreased by this product.

a) Let $P(x)$ be the income of the company if there are x passengers accepted on a flight. Find a fromula for the function $P: x \mapsto P(x)$. Find also the domain of the function P.

b) How many passengers shall the company carry on a flight in order to maximize its income on the given flight and how much is the maximal income?

a)	6 points	
b)	7 points	
T.:	13 points	

4. According to a survey conducted among those possessing internet connection 17% of them does shopping on the internet and 33% of them is downloading softwares regularly. It was also found out that 14% of the internet users make use of both services. Find the probablities of the following events, respectively.

   a) A randomly selected internet user does not do shopping on the net.

   b) A randomly selected internet user does shopping on the net or she is regularly downloading softwares. (She may make use of both services.)

   c) A randomly selected internet user neither does shopping on the net nor downloads softwares from the net.

   d) Among three randomly selected internet users there is no one who does shopping on the net. (Selection is by replacement here.)

a)	3 points	
b)	4 points	
c)	3 points	
d)	4 points	
T.:	14 points	

II.

You are supposed to answer any four of the questions no. 5-9. The number of the question not attempted should be entered into the empty square on sheet no. 3.

5. There were more than 400 students attending a certain school at the end of the academic year but their number was certainly less than 430. At the end of the academic year the average of female students was 4.21, that of male students was 4.01, finally, the average of the total student population was 4.12. (Each average is accurate.) How many students did attend this school at the end of the academic year?

T.:	16 points	

You are supposed to answer any four of the questions no. 5-9. The number of the question not attempted should be entered into the empty square on sheet no. 3.

6. There was a ditch dug out of a plane ground. Being 6 m deep it is narrowing downwards; its sidewalls are planar quadrilaterals. The base of the ditch is horizontal flat. Its opening is a square of size 8 × 8 m, two of its opposite sidewalls are vertical, while the other two are making angles of 75° and 60° with horizontal ground level, respectively, in such a way that the angle of the planes of these slant sidewalls is 45°.

    a) Sktech, in a neat diagram, the ditch's cross section, by a plane that is perpendicular to the slant sidewalls (and hence also to the ground level). Indicate also the data on your diagram.

    b) How many m^3 soil had to be dug out while completing this ditch. The result should be given to the nearest m^3.

a)	4 points	
b)	12 points	
T.:	16 points	

You are supposed to answer any four of the questions no. 5-9. The number of the question not attempted should be entered into the empty square on sheet no. 3.

7. The class 12.A was offered five tickets for the finals of the waterpolo tournament. Each of the thirty students of the class would go willingly, even if twelve of them will have revision class during the finals. They decide to organize a draw to find the five lucky ones: each of the 30 names is written on a slip of paper, respectively, and five of them are drawn.

    a) What is the probability that there are exactly two among the five hence selected who have revision class? The answer should be given as a decimal fraction.

    b) We got to know that there happen to be students among the five selected who actually have a revision class. Given this information what is the probability that there are exactly two among the five selected students who have revision class?

After the finals the five lucky spectators made some comments on the match:
    A: The loser team has scored more than 4 times.
    B: The winner team has scored 3 more than the other team.
    C: The total number of goals was greater than 10 but smaller than 28.
    D: The total number of goals is a prime number.
    E: The number of goals scored by the loser team is also a prime number.

    c) Is it possible, based on the above information, to deduce the result of the finals assuming that each student told the truth?

a)	4 points	
b)	7 points	
c)	5 points	
T.:	16 points	

You are supposed to answer any four of the questions no. 5-9. The number of the question not attempted should be entered into the empty square on sheet no. 3.

8. The sequences $\{a_n\}$, $\{b_n\}$ and $\{c_n\}$ are geometric progressions formed by integer numbers, each. About the respective common ratios and certain terms of the progressions we are given the following information:

(1) a_1, b_1 and c_1 are, in this order, the consecutive terms of a geometric progression whose common ratio is equal to 2;

(2) the common ratios of the progressions $\{a_n\}$, $\{b_n\}$ and $\{c_n\}$ are, in this order, the consecutive terms of an arithmetic progression whose common difference is equal to 1;

(3) $a_2 + b_2 + c_2 = 24$;

(4) $c_1 + c_2 + c_3 = 84$.

Find the first three terms of the geometric progressions $\{a_n\}$, $\{b_n\}$ and $\{c_n\}$, repectively.

T.:	16 points	

You are supposed to answer any four of the questions no. 5-9. The number of the question not attempted should be entered into the empty square on sheet no. 3.

9. The equation of a parabola is $y = x^2 - 4x - 12$. Denote its vertex by C, and its intersections with the x-axis by A and B, respectively.
   a) Calculate the inradius of the triangle ABC.
   b) Find the ratio between the area of the triangle ABC and that of the closed region bounded by the parabola and the x-axis.

a)	8 points	
b)	8 points	
T.:	16 points	

	number of question	maximal score	score	maximal score	score
PART I	1.	12		**51**	
	2.	12			
	3.	13			
	4.	14			
PART II.		16		**64**	
		16			
		16			
		16			
	← problem not chosen				
Score of the written component				**115**	

_____   _____
          date   examiner

	score attained rounded to the **nearest integer** (elért pontszám **egész számra** kerekítve)	Integer score entered in the program (programba beírt **egész** pontszám)
Paper I/(I. rész)		
Paper II/(II. rész)		

_____   _____
   teacher/(javító tanár)   registrar/(jegyző)

_____   _____
    date /(dátum)   date /(dátum)